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Quasilocality of joining/splitting strings from coherent states

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Abstract: Using the coherent state formalism we calculate matrix elements of the one-loop non-planar dilatation operator of $\mathcal{N} = 4$ SYM between operators dual to folded Frolov-Tseytlin strings and observe a curious scaling behavior. We comment on the *qualitative* similarity of our matrix elements to the interaction vertex of a string field theory. In addition, we present a solvable toy model for string splitting and joining. The scaling behaviour of the matrix elements suggests that the contribution to the genus one energy shift coming from semi-classical string splitting and joining is small.

Keywords: AdS-CFT Correspondence, Bosonic Strings.

Contents

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1. Introduction

Integrability has played a key role in recent years exploration of planar $\mathcal{N} = 4$ SYM [1−3] as well as non-interacting type IIB string theory on $AdS_5 \times S^5$ [4, 5], tied together by the AdS/CFT correspondence [6]. Whereas integrability is expected to break down beyond the planar/non-interacting limit — most clearly demonstrated by the lift of degeneracies of anomalous dimensions in the gauge theory [2] — the AdS/CFT correspondence could still be valid [6, 7]. Lacking the framework of integrability, tests of the AdS/CFT correspondence beyond the planar limit have proved difficult. Even in the BMN limit [8] where the free string theory can actually be quantized no conclusive tests exist. For an up to date review, see [9]. The gauge theory calculations, although described efficiently by a quantum mechanical Hamiltonian [10], are plagued by huge degeneracy problems [11]. The string theory computations on their side suffer from the existence of several competing proposals for the three string vertex of light cone string field theory and from the necessity of truncating the vertex to a subset of decay channels. Although the BMN limit seems to be the most tractable one as regards the analysis of the non-planar sector of the theories it might be instructive to perform the analysis in other limits as well. A limit which has been instrumental in the investigation of the planar/non-interacting case is the Frolov-Tseytlin limit [12]. A first step in the direction of extending the analysis of this limit to the non-planar/interacting situation was taken in [13] where the decay of a folded Frolov-Tseytlin string [14] was described using semi-classical methods. Based on the investigations performed it was argued that the integrability observed for the free string may survive in certain decay channels. In the present paper we attack the non-planar Frolov-Tseytlin limit from the gauge theory side. Using a coherent state approach we calculate matrix elements of the one-loop non-planar dilatation generator of $\mathcal{N}=4$ SYM between operators dual to folded Frolov-Tseytlin strings rotating on $S^3 \subset S^5 \subset AdS_5 \times S^5$.

We begin in section 2 by presenting the form of the one-loop non-planar dilatation operator in the SU(2) sector of $\mathcal{N}=4$ SYM. Subsequently, in section 3 we review the coherent state description of the operator dual to the folded Frolov-Tseytlin string. Section 4 deals with the calculation of matrix elements for the gauge theory equivalent of string joining and string splitting. In section 5 we describe a solvable toy model for the decay of the folded string which unfortunately is only a very crude approximation to the actual model. Finally, section 6 contains a discussion.

2. The one-loop non-planar dilatation operator

We consider the SU(2) sector of $\mathcal{N}=4$ SYM consisting of multi-trace operators built from the two complex scalar fields Z and Φ. In this sub-sector the complete one-loop dilatation operator can be expressed as [15, 2]

$$H = -\frac{g_{\rm YM}^2}{8\pi^2}\mathrm{Tr}[\Phi, Z][\check{\Phi}, \check{Z}], \qquad \check{Z} = \frac{\delta}{\delta Z}, \tag{2.1}$$

or equivalently [16, 13]:

$$H = H_P + H_{NP}, \tag{2.2}$$

where

$$H_P = \lambda \sum_k (1 - P_{k,k+1}), \qquad \lambda = \frac{g_{\rm YM}^2 N}{8\pi^2}, \tag{2.3}$$

and

$$H_{NP} = \frac{\lambda}{N} \sum_{k,\, l\neq k+1} (1 - P_{k,l})\ \Sigma_{k+1,l}, \tag{2.4}$$

with H_P being the planar part and H_{NP} the non-planar one. Here the indices refer to the position of the fields inside the operator on which H acts. The indices are periodically identified as dictated by the trace structure of the operator. The operator $P_{k,l}$ simply interchanges indices k and l. Furthermore, if one represents an operator as a set of fields plus a permutation element giving the ordering of the fields, then $\Sigma_{k,l}$ is just the transposition $\sigma_{k,l}$ applied on this permutation [16]. A useful way of describing the effect of having acted with $\Sigma_{k,l}$ on a chain of fields is the following (see also figure 1):

The site that was going to k goes to l and vice versa.

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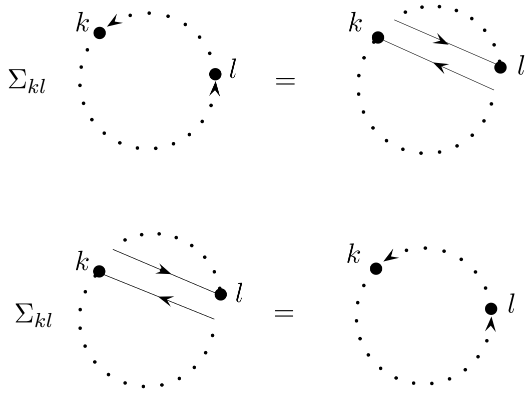

Figure 1: Splitting and joining of chains by Σ_{kl}.

3. Folded string duals using coherent states

3.1 The Frolov-Tseytlin folded string

We wish to consider operators dual to the folded Frolov-Tseytlin string spinning on $S^3 \subset S^5 \subset AdS_5 \times S^5$ with two large angular momenta (J_1, J_2). More precisely, we consider the limit $J_1, J_2 \to \infty$ with $\frac{J_1}{J_2}$ finite. A semi-classical analysis of the string in question yields that its energy has the following expansion [14]

$$E = J\left(1 + \frac{\lambda}{J^2}\,\mathcal{E}_0 + \frac{\lambda^2}{J^4}\,\mathcal{E}_0^{(1)} + \ldots\right), \qquad J = J_1 + J_2, \tag{3.1}$$

with the gauge coupling constant λ appearing via the AdS/CFT dictionary $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ [6] and where we also assume that $\frac{\lambda}{J^2}$ is finite. The term of linear order in λ is found to be

$$\mathcal{E}_0 = 16\,K(m)\left(E(m) - (1-m)K(m)\right), \tag{3.2}$$

where $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and the second kind respectively.[1] The parameter m is determined by

$$\frac{J_2}{J} = 1 - \frac{E(m)}{K(m)}. \tag{3.3}$$

The gauge theory dual of the folded Frolov-Tseytlin string is a complicated linear combination of single trace operators each containing J_1 Φ's and J_2 Z's [14, 17]. It is characterized by being an eigenstate of the one-loop planar dilatation operator, H_P, cf. eq. (2.3), with eigenvalue given by $\frac{\lambda}{J}\mathcal{E}_0$. A more efficient way of describing the dual is by means of SU(2) spin-1/2 coherent states. To introduce these, let us denote the two normalized eigenstates of S_z by $|\uparrow\rangle$ and $|\downarrow\rangle$. These states have the inner product

$$\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1,$$
$$\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0.$$

[1] Here and in the following we use the Mathematica definition of elliptic functions and integrals.

The relevant coherent states then take the form

$$|\vec{n}\rangle = \cos\theta\,|\!\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\,\varphi}\sin\theta\,|\!\downarrow\rangle\,, \tag{3.4}$$

where the angles $\theta \in [0, \frac{\pi}{2}]$ and $\varphi \in [0, 2\pi]$ parametrize a unit three vector $\vec{n}$ by

$$\vec{n} = (\cos 2\theta \sin\varphi, \sin 2\theta \sin\varphi, \cos\varphi)\,. \tag{3.5}$$

The folded string dual can now be described as a state of a SU(2) spin chain of length J having a coherent state vector at each site [18]. Without loss of generality we will take J to be a multiple of four, in order for the spin chain to reflect as closely as possible the symmetries of the folded string (the entire string profile follows from its definition on a quarter period). The state representing the string thus reads

$$|\mathbf{n}\rangle = \left|\vec{n}_{-\frac{J}{2}}\right\rangle \otimes \left|\vec{n}_{-\frac{J}{2}+1}\right\rangle \cdots \otimes \left|\vec{n}_{\frac{J}{2}}\right\rangle, \tag{3.6}$$

where obviously

$$|\vec{n}_k\rangle = \cos\theta_k\,|\!\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\,\varphi_k}\sin\theta_k\,|\!\downarrow\rangle\,. \tag{3.7}$$

Here the planar energy of the string is obtained as $\frac{\lambda}{J}\mathcal{E}_0 = \langle\mathbf{n}|H_P|\mathbf{n}\rangle$. In the long wavelength limit where θ_k and φ_k vary only slowly and where $J \to \infty$, which exactly corresponds to the Frolov-Tseytlin limit, one can replace the θ_k and φ_k by continuous functions $\theta_k \to \theta(\sigma = \frac{k}{J})$ and $\varphi_k \to \varphi(\sigma = \frac{k}{J})$ and one can derive an effective sigma model action describing the model. The cyclicity property of the gauge theory operator translates into the requirement of vanishing momenta in the σ direction, which reads

$$\mathcal{P}_\sigma = -\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\theta)\partial_\sigma\varphi\,\mathrm{d}\sigma = 0\,. \tag{3.8}$$

The equations of motion following from the above mentioned action permit a solution exactly describing the folded Frolov-Tseytlin string dual. For this solution one has

$$\theta'^2 - \frac{\omega}{2\lambda}\,(\cos 2\,\theta - \cos 2\,\theta_0) = 0, \qquad \varphi = \omega\,t, \tag{3.9}$$

which in particular is seen to fulfill the relation (3.8). The angle θ can be expressed in terms of the Jacobi sn function

$$\sin\theta(\sigma) = \sin\theta_0\ \mathrm{sn}\left(J\sqrt{\frac{\omega}{\lambda}}\,\sigma\,\middle|\,\sin^2\theta_0\right), \tag{3.10}$$

where the following relation between θ_0 and ω must hold for the string to be closed and folded exactly once

$$J\sqrt{\frac{\omega}{\lambda}} = 4K(m), \qquad m = \sin^2(\theta_0). \tag{3.11}$$

The angular variable $\theta(\sigma)$ obviously varies in the interval $[-\theta_0, \theta_0]$. For any given θ_0 one has (or can impose) the following identifications, see figure 2 .

$$\forall n \in \mathbb{N},\ \forall z \in \mathbb{R}, \qquad \theta(z+n) = \theta(z)\,, \quad \theta\left(\frac{1}{2} - z\right) = \theta(z)\,. \tag{3.12}$$

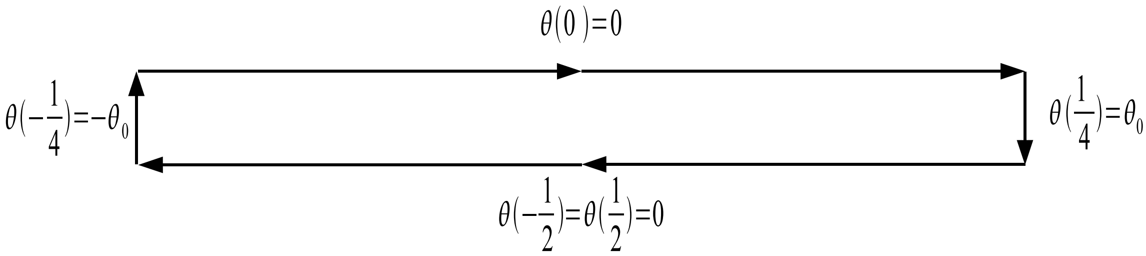

Figure 2: Different values of $\theta = \theta(\sigma)$ along the string.

In this formulation the one-loop anomalous dimension of the gauge theory operator is given by [18]

$$\mathcal{E}_0 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \theta'(\sigma)^2 \, \mathrm{d}\sigma, \tag{3.13}$$

and

$$\frac{J_2}{J} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sin^2 \theta(\sigma) \, \mathrm{d}\sigma. \tag{3.14}$$

which are easily seen to reproduce eqs. (3.2) and (3.3).

3.2 Coherent state strings

The coherent state vectors $|\mathbf{n}\rangle$ single out the endpoint of the folded string — a property which is not natural from the dual gauge theory perspective as the dual operator must be cyclically symmetric.[2] This, in particular, becomes an issue when we wish to calculate matrix elements between multi-cut states, cf. section 4.

We can ensure cyclicity of the state by averaging over cyclic translations:

$$|\mathbf{n}\rangle\rangle = \sum_{k=1}^{n} \prod_{i=1}^{L_m} |\overrightarrow{n_{i+k}}\rangle \, . \tag{3.15}$$

These averaged states, properly normalized, will now represent our string states. The inner product is defined as follows. Given two vectors $|\mathbf{n}\rangle = \prod_{i=1,L_n} |\overrightarrow{n_i}\rangle$ and $|\mathbf{m}\rangle = \prod_{j=1,L_m} |\overrightarrow{m_j}\rangle$ one has

$$\langle \mathbf{m} | \mathbf{n} \rangle = \delta_{L_m, L_n} \prod_{i=1}^{L_m} \langle \overrightarrow{m_i} \, | \, \overrightarrow{n_i} \rangle \, , \tag{3.16}$$

from which the definition of $\langle\langle \mathbf{m} | \mathbf{n} \rangle\rangle$ follows.

4. Matrix elements of H_{NP}

With our new states we have

$$\frac{\lambda}{J} \mathcal{E}_0 = \frac{\langle\langle \mathbf{n} | H_P | \mathbf{n} \rangle\rangle}{\langle\langle \mathbf{n} \, | \, \mathbf{n} \rangle\rangle}. \tag{4.1}$$

[2]As mentioned above, in the coherent state framework cyclicity manifests itself via the equation (3.8).

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We would now like to calculate matrix elements of the one-loop non-planar dilatation operator between coherent state vectors representing folded Frolov-Tseytlin strings. It is obvious that acting on a coherent state vector $|\mathbf{n}\rangle$ with H_{NP} gives rise to a splitting of a one-string dual into a two-string dual. Similarly, acting with H_{NP} on a direct product two coherent state vectors $|\mathbf{n}\rangle$ and $|\mathbf{m}\rangle$ can produce a one-string dual from a two-string dual. In a more traditional gauge theory language H_{NP} gives rise to trace splitting and trace joining. The matrix elements of the non-planar dilatation operator contain information about the genus one correction to the energy of Frolov-Tseytlin strings. It is obvious, however, that if we would try to determine this energy correction by considering H_{NP} a perturbation of H_P we would have to make use of degenerate perturbation theory. For instance, if we start from a coherent state vector $|\mathbf{n}\rangle$ of energy $\mathcal{E}_0$ as given by eq. (4.1), cut it vertically once and close the open ends we obtain another state which up to $1/J$ corrections is an eigenstate with the same energy. The same is true if we make l vertical cuts where $l \ll J$, see figure 3. We could also cut with some, not too large, skewness and still obtain a degenerate state. However, we will restrict ourselves to straight cut states since in the continuum limit small skewness should not matter and large skewness takes us out of the sub-space of degenerate states. We notice that since $\varphi = \omega t$ is constant along the string the inner product between two coherent states reduces to

$$\langle \vec{n}_1 | \vec{n}_2 \rangle = \cos(\theta_1 - \theta_2), \tag{4.2}$$

which implies that we do not need to worry about φ_i at all and can consistently set $\varphi_i = 0$.

4.1 Normalization of states

Let us denote by $|\emptyset\rangle$ the complete (uncut) folded string dual, i.e.

$$|\emptyset\rangle \equiv \prod_{i=-J/2}^{J/2} |\vec{n}_i\rangle \tag{4.3}$$

with

$$|\vec{n}_i\rangle = \cos\theta\left(\frac{i}{J}\right)|\uparrow\rangle + \sin\theta\left(\frac{i}{J}\right)|\downarrow\rangle, \qquad -\frac{J}{2} < i < \frac{J}{2}, \tag{4.4}$$

and with $\theta(x)$ the function given in equation (3.10). Furthermore, let us denote by $|x_1, \ldots, x_l\rangle$ the state obtained from (4.3) by cutting it vertically at the points $x_1, x_2, \ldots x_l$, (see figure 3)

$$\begin{aligned} &|x_1, \cdots, x_l\rangle \equiv \\ &\left| \prod_{i=-J/4}^{x_1 J} \vec{n}_i \prod_{i=-J/4}^{x_1 J} \vec{n}_{(x_1-\frac{1}{4})J-i} \right\rangle \otimes \left| \prod_{k=x_1 J+1}^{x_2 J} \vec{n}_i \prod_{k=x_1 J+1}^{x_2 J} \vec{n}_{(x_1+x_2)\,J+1-i} \right\rangle \otimes \\ &\cdots \otimes \left| \prod_{k=x_l J+1}^{J/4} \vec{n}_i \prod_{k=x_l J+1}^{J/4} \vec{n}_{(x_l+\frac{1}{4})\,J+1-i} \right\rangle, \end{aligned} \tag{4.5}$$

where

$$-\frac{1}{4} < x_i < \frac{1}{4}, \qquad l \ll J, \qquad x_{j+1} - x_j \sim \mathcal{O}(J). \tag{4.6}$$

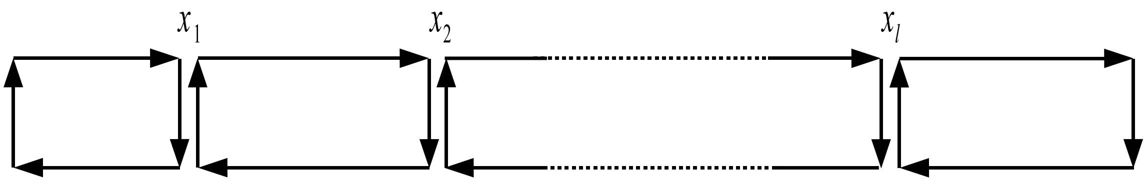

Figure 3: A cut state $|x_1, \cdots, x_l\rangle$.

In order to determine the norm of such a state, we first consider a single piece of string, extending between the points x and y and compute the inner product $\langle\,|\,\rangle$ between this piece and the piece which appears from it by shifting each of its coherent state vectors a distance δ.

$$\mathcal{A}_{x,y,\delta} \equiv \langle \prod_{i=x\,J}^{y\,J} \vec{n}_{i-\delta\,J} \,|\, \prod_{i=x\,J}^{y\,J} \vec{n}_i \rangle. = \prod_{i=0}^{(y-x)\,J} \langle \vec{n}_{(x-\delta)\,J+i} \,|\, \vec{n}_{x\,J+i} \rangle. \tag{4.7}$$

For fixed δ, it is clear that $\mathcal{A}_{x,y,\delta}$ goes exponentially to zero as J goes to infinity. It is therefore sufficient to study the behavior of $\mathcal{A}_{x,y,\delta}$ for small δ:

$$\begin{aligned}\mathcal{A}_{x,y,\delta} &\approx \exp\left[J \int_x^y \log\left[\cos\left[\theta(z-\delta)-\theta(z)\right]\right]\,\mathrm{d}z\right] \\ &\approx \exp\left[-J\frac{\delta^2}{2}\int_x^y \theta'(z)^2\,\mathrm{d}z\right] \\ &\approx \exp\left[-J\frac{\delta^2}{2}\,\mathcal{E}_{x,y}\right],\end{aligned} \tag{4.8}$$

where $\mathcal{E}_{x,y}$ is given by

$$\begin{aligned}\mathcal{E}_{x,y} &\equiv \int_x^y \theta'(z)^2\,\mathrm{d}z \qquad (4.9)\\ &= 4\,K(m)\Big(\mathrm{E}\left[\mathrm{am}\left(4\,K\,y|m\right)\right]-\mathrm{E}\left[\mathrm{am}\left(4\,K\,x|m\right)\right]-4\,K(m)\,\left(1-m\right)\,\left(y-x\right)\Big).\end{aligned}$$

Notice that the planar energy of the folded string stretching between x and y is $2\mathcal{E}_{x,y}$ and in particular by definition $\mathcal{E}_0 = \mathcal{E}_{-\frac{1}{2},\frac{1}{2}}$. It is then easy to find the square of the norm of the string with no cuts at leading order in J by integrating over all possible[3] δ:

$$\langle\!\langle\, \emptyset | \emptyset \,\rangle\!\rangle = J^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} \exp\left[-J\frac{\mathcal{E}_0}{2}\,\delta^2\right]\mathrm{d}\delta = J\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_0}}\ . \tag{4.10}$$

One of the factors of J comes from the fact that one can simultaneously make the same cyclic translation of the bra and the ket without changing anything. The second factor of J comes from the summation over nontrivial *relative* translations, and the substitution

[3]Since we assume that $x_{j+1} - x_j \sim \mathcal{O}(J)$, the integration range of such a Gaussian integral can always be taken to be $]-\infty, +\infty[$ when $J \to \infty$.

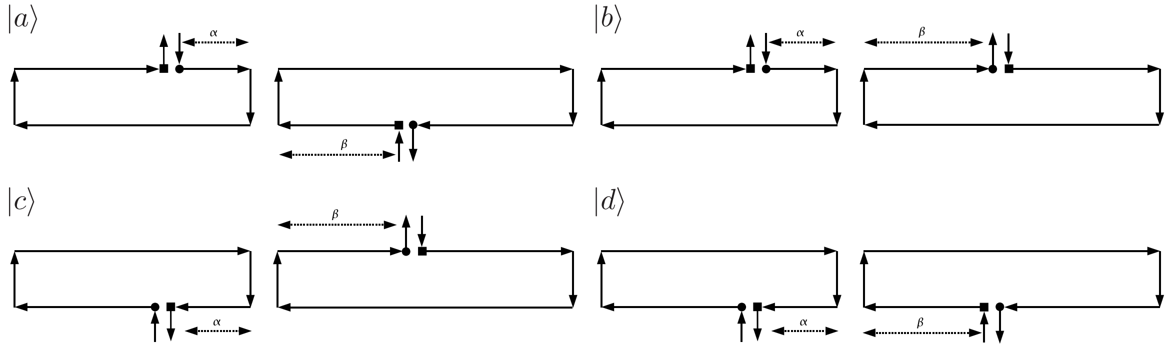

Figure 4: Possible joinings of two bits. Sites at the squares (circles) are linked after joining and then antisymmetrized.

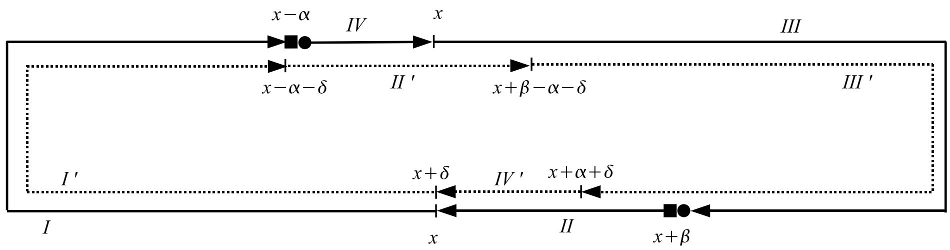

Figure 5: Overlaps between the bra $\langle\emptyset|$ (dotted lines) and the ket $|a\rangle$ (continuous lines). Arguments for $\theta(x)$ are given at the relevant points. More precisely, the bra reads $\langle I'\ II'\ III'\ IV'|$ and the ket $|I\ II\ III\ IV\rangle$: in this figure, it is the function $\theta(x)$ which is continuous along the loop while the sequence inside the ket is discontinuous.

of a continuous integral for the discrete sum in the large J limit. For each smaller string in (4.5), one will get a similar factor so that

$$\langle\langle x_1, x_2, \cdots, x_l | x_1, x_2, \cdots, x_l \rangle\rangle = \prod_{i=0}^{l} \frac{l_i\, J\, (\pi\, J)^{\frac{1}{2}}}{\sqrt{\mathcal{E}_{x_i, x_{i+1}}}}, \tag{4.11}$$

where $x_0 \equiv -\frac{1}{4}$, $x_{l+1} \equiv \frac{1}{4}$ and $l_i \equiv 2\,(x_{i+1} - x_i)$.

Here, we neglected the contributions coming from the "corners" of the string pieces where the overlap is not anymore between $\theta(z-\delta)$ and $\theta(z)$ as in (4.8). This is justified because the relevant shifts $\delta\, J$ are much smaller than the length of the pieces we consider.

4.2 Matrix elements for string joining

We compute in this section the matrix element $\langle\langle\emptyset|H_{\rm NP}|x\rangle\rangle$. To begin with we consider non-cyclic states.

There are in total four ways to join a two-piece state, giving rise to the four different states $|a\rangle$, $|b\rangle$, $|c\rangle$ and $|d\rangle$ as shown in figure 4. The eventual use of the cyclic states $|\,\rangle\rangle$ is essential here since the notion of the endpoint of the string becomes ambiguous. By reflection symmetry, states $|a\rangle$ and $|c\rangle$ give the same expectation values, and so do states $|b\rangle$ and $|d\rangle$. We will start with state $|a\rangle$. The corresponding overlaps are shown in figure 5.

As in the previous section, we denote by δ the shift given to $\langle\emptyset|$ and by $\langle I'_\delta|$, $\langle II'_\delta|$, $\langle III'_\delta|$, $\langle IV'_\delta|$ its corresponding δ-shifted pieces (see figure 5). We also define the planar energies of the first and second spin chain bits respectively by

$$\mathcal{E}_1 \equiv \mathcal{E}_{-\frac{1}{2}-x,x} = 2\,\mathcal{E}_{-\frac{1}{4},x} \quad \text{and} \quad \mathcal{E}_2 \equiv \mathcal{E}_{x,\frac{1}{2}-x} = 2\,\mathcal{E}_{x,\frac{1}{4}}\ .$$

The identity $\mathcal{E}_0 = \mathcal{E}_1 + \mathcal{E}_2$ is satisfied by construction. First, let us assume that $\beta \geq \alpha$. We have

$$\langle\langle\emptyset|a\rangle\rangle = \sum_\delta \mathcal{F}_{\alpha,\beta,\delta}\,\langle I'_\delta|I\rangle\,\langle II'_\delta|II\rangle\,\langle III'_\delta|III\rangle\,\langle IV'_\delta|IV\rangle\,,$$

where anti-symmetrization effects at the joining sites are taken into account through the $\mathcal{F}_{\alpha,\beta,\delta}$ factor.

In order to do the computation, we expand as follows

$$\log\left[\cos\left[\theta(z-\epsilon)-\theta(z)\right]\right] = -\frac{\epsilon^2}{2}\,\theta'(z)^2 + \frac{\epsilon^3}{2}\,\theta'(z)\theta''(z) + \mathcal{O}(\epsilon^4), \tag{4.12}$$

and make use of the identities (3.12) for $\theta(z)$. It is important to stress that the expansion we will use for the integrands strongly depends on the range of integration. For long range integrations, e.g $\int_{-\frac{1}{2}-x}^{x} f(z,x,\alpha,\beta,\delta)\mathrm{d}z$, we expand for small α,β,δ's only. For short range integrations, e.g $\int_0^\beta f(z,x,\alpha,\beta,\delta)\mathrm{d}z$, we also expand for small z's.

One then gets

$$\begin{aligned}
\langle I'_\delta|I\rangle &= \langle \prod_{i=(-\frac{1}{2}-x)\,J}^{(x-\alpha)\,J} \vec{n}_{i-\delta\,J}\,, \prod_{i=(-\frac{1}{2}-x)\,J}^{(x-\alpha)\,J} \vec{n}_{i}\rangle \\
&\approx \exp\left[J\int_{-\frac{1}{2}-x}^{x-\alpha} \log\left[\cos\left[\theta(z-\delta)-\theta(z)\right]\right]\,\mathrm{d}z\right] \\
&\approx \exp\left[J\int_{-\frac{1}{2}-x}^{x}\left(-\frac{\delta^2}{2}\,\theta'(z)^2 + \frac{\delta^3}{2}\,\theta'(z)\theta''(z)\right)\mathrm{d}z + J\frac{\delta^2\alpha}{2}\theta'(x)^2\right] \\
&\approx \exp\left[-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2 + J\frac{\delta^2\alpha}{2}\theta'(x)^2\right]\,,
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
\langle II'_\delta|II\rangle &= \langle \prod_{i=0}^{\beta\,J} \vec{n}_{(x+\beta-\alpha-\delta)\,J-i}\,, \prod_{i=0}^{\beta\,J} \vec{n}_{x\,J+i}]\rangle \\
&\approx \exp\left[-\frac{1}{2}\,\theta'(x)^2\,J\int_0^\beta (2\,z+\alpha-\beta+\delta)^2\mathrm{d}z\right] \\
&\approx \exp\left[-\frac{1}{6}\,J\,\beta\left(\beta^2+3\,(\alpha+\delta)^2\right)\,\theta'\,(x)^2\right]\,,
\end{aligned} \tag{4.14}$$

$$\begin{aligned}
\langle III'_\delta|III\rangle &= \langle \prod_{i=x\,J}^{(\frac{1}{2}-x-\beta)\,J} \vec{n}_{(\beta-\alpha-\delta)\,J+i}\,, \prod_{i=x\,J}^{(\frac{1}{2}-x-\beta)\,J} \vec{n}_{i}\rangle \\
&\approx \exp\left[J\int_x^{\frac{1}{2}-x}\left(-\frac{(\beta-\alpha-\delta)^2}{2}\,\theta'(z)^2 + \frac{(\beta-\alpha-\delta)^3}{2}\,\theta'(z)\theta''(z)\right)\mathrm{d}z.\right. \\
&\qquad\left. +J\frac{(\beta-\alpha-\delta)^2\beta}{2}\theta'\left(\frac{1}{2}-x\right)^2\right]
\end{aligned} \tag{4.15}$$

$$
\begin{aligned}
&\approx \exp\left[-\frac{1}{2}\,J\,(\beta-\alpha-\delta)^2\;\mathcal{E}_2 + J\,\frac{(\beta-\alpha-\delta)^2\,\beta}{2}\,\theta'(x)^2\right]\,, && (4.16)\\
\langle IV'_\delta | IV\rangle &= \langle \prod_{i=0}^{\alpha J} \overrightarrow{n}_{(x+\delta)\,J+i}, \prod_{i=0}^{\alpha J} \overrightarrow{n}_{x\,J-i}\rangle \\
&\approx \exp\left[-\frac{1}{2}\,\theta'(x)^2\,J\int_0^\alpha (2\,z+\delta)^2\,\mathrm{d}z\right] \\
&\approx \exp\left[-\frac{1}{6}\,J\,\alpha\,\left(4\,\alpha^2+6\,\alpha\,\delta+3\,\delta^2\right)\,\theta'\,(x)^2\right]\,. && (4.17)
\end{aligned}
$$

The four overlaps in total give the contribution

$$
\begin{aligned}
\exp\Bigg[&-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2 - \frac{1}{2}\,\mathcal{E}_2\,J\,(\beta-\alpha-\delta)^2 \\
&-\frac{1}{3}\,J\,\theta'(x)^2\,\left(-\alpha^3+3\,\alpha^2\,\beta+2\,\beta^3-3\,\left(\beta^2+\alpha^2\right)\,(\beta-\alpha-\delta)\right)\Bigg],
\end{aligned}
$$

and one can see that the dominant region will be around $\delta\approx 0$ and $\beta\approx\alpha$, so that the leading term in $\frac{1}{J}$ will be given by taking the following approximation for the exponential:

$$
\exp\left[-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2 - \frac{1}{2}\,\mathcal{E}_2\,J\,(\beta-\alpha)^2\;-\frac{4}{3}\,J\,\alpha^3\,\theta'(x)^2\right].
$$

We should now compute $\mathcal{F}_{\alpha,\beta,\delta}$ near these values of α, β and δ. One gets[4]

$$
\begin{aligned}
\mathcal{F}_{\alpha,\alpha,0} &= \Big\langle \overrightarrow{n}_{(x-\alpha-\delta)\,J}\,,\,\overrightarrow{n}_{(x-\alpha)\,J}\Big\rangle\,\Big\langle \overrightarrow{n}_{(x-\alpha-\delta)\,J+1}\,,\,\overrightarrow{n}_{(x+\beta)\,J}\Big\rangle \\
&\qquad\times\,\Big\langle \overrightarrow{n}_{(x+\alpha+\delta)\,J+1}\,,\,\overrightarrow{n}_{(x+\beta)\,J+1}\Big\rangle\,\Big\langle \overrightarrow{n}_{(x+\alpha+\delta)\,J}\,,\,\overrightarrow{n}_{(x-\alpha)\,J+1}\Big\rangle\Bigg|_{\substack{\delta=0\\ \beta=\alpha}} \\
&\approx \frac{4}{J}\alpha\,\theta'(x)^2\,. && (4.18)
\end{aligned}
$$

The case $\alpha>\beta$ gives the same result up to the exchange $\alpha\leftrightarrow\beta$. Furthermore, translating the result to cyclic states implies multiplying by $l_1\,l_2\,J^2$. Finally, using the normalization factor $\mathcal{N}=\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_0}}\,l_0\,J\right)^{\frac{1}{2}}\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_1}}\,l_1\,J\right)^{\frac{1}{2}}\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_2}}\,l_2\,J\right)^{\frac{1}{2}}$, one then gets at leading order in $\frac{1}{J}$:

$$
\begin{aligned}
\sum_{\alpha,\beta}\langle\!\langle\emptyset|a\rangle\!\rangle &\approx \frac{2}{\mathcal{N}}\,\frac{4}{J}\,\theta'(x)^2J^5\,l_1\,l_2\int_0^\infty\mathrm{d}\beta\int_0^\beta\mathrm{d}\alpha\int_{-\infty}^\infty\mathrm{d}\delta\;\;e^{-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2-\frac{1}{2}\,\mathcal{E}_2\,J\,(\beta-\alpha)^2\;-\frac{4}{3}\,J\,\alpha^3\,\theta'(x)^2}\;\alpha \\
&\approx \frac{4\,\Gamma\left(\frac{2}{3}\right)}{3^{1/3}}\,K^{2/3}\,m^{1/3}\,\mathrm{cn}\,(4\,K\,x|m)^{2/3}\,\left(\frac{l_1\,l_2}{l_0}\right)^{1/2}\left(\frac{2\,\pi\,\mathcal{E}_0}{\mathcal{E}_1\,\mathcal{E}_2}\right)^{\frac{1}{4}}\,J^{1/12}\,. && (4.19)
\end{aligned}
$$

Note that although β should be in the interval $[0,\frac{1}{4}-x]$ and δ in the interval $[-\frac{1}{2},\frac{1}{2}]$, integrating in both cases till infinity will not change the leading $\frac{1}{J}$ behavior as the integrand converges exponentially to zero for $\alpha\,J^{1/3}\gg 1$, $\beta\,J^{1/3}\gg 1$ and $\delta\,J^{1/2}\gg 1$.

[4]We use the notation $f(A,B)\,g(C,D)=(f(A,B)-f(B,A))\;g(C,D)+f(A,B)\;(g(C,D)-g(D,C))$.

A similar computation shows that $\langle\langle\emptyset|b\rangle\rangle$ and $\langle\langle\emptyset|d\rangle\rangle$ are of order $J^{-1/4}$ and therefore can be neglected compared to the $J^{1/12}$ behavior found here. Thus, one obtains at leading order in $\frac{1}{J}$

$$\langle\langle\emptyset|H_{\mathrm{NP}}|x\rangle\rangle = \frac{8\,\Gamma\left(\frac{2}{3}\right)}{3^{1/3}}\,K^{2/3}\,m^{1/3}\,\mathrm{cn}\,(4\,K\,x|m)^{2/3}\left(\frac{l_1\,l_2}{l_0}\right)^{1/2}\left(\frac{2\,\pi\,\mathcal{E}_0}{\mathcal{E}_1\,\mathcal{E}_2}\right)^{\frac{1}{4}} J^{1/12}\,. \tag{4.20}$$

It is straightforward to generalize this result to an arbitrary number of cuts where the joining takes place at position x_i. It is in order to facilitate this generalization that we have explicitly kept the parameter l_0 although in our case we have $l_0 = 1$. We observe the occurrence of the factor $(\mathcal{E}_1\,\mathcal{E}_2)^{-1/4}$ which diverges when x approaches the endpoints of the string. In this situation we can thus not trust the semi-classical analysis (and hence the overall J-scaling).

4.3 Matrix elements for string splitting

From the calculations in the last section, we learn which approximations we are allowed to do in order to keep only the leading order in $\frac{1}{J}$. First, the terms that arise from the cyclicity of the traces are long range terms: they appear through δ-shifts over a whole piece of spin chain and consequently will give in the exponential a square term times minus the planar energy of the considered piece, times J. This is what happened in equations (4.13) and (4.16). Conversely, terms which are integrated on short intervals will appear in the exponential starting at the cubic order (see equations (4.14) and (4.17)). This allows for the following approximations that will not change the leading $\frac{1}{J}$ term after all integrations:

1. When computing overlaps over long range parts, it is not necessary to take into account small parameters at the endpoints of the integration. For example, taking $\int_{-\frac{1}{2}-x}^{x} \mathrm{d}z$ instead of $\int_{-\frac{1}{2}-x}^{x-\alpha} \mathrm{d}z$ in (4.13) would not have changed the final result.

2. When computing overlaps over short range parts, one can do as if the shifts appearing in the long range terms were equal to zero.

We can now compute expectation values such as $\langle\langle x|H_{\mathrm{NP}}|\emptyset\rangle\rangle$. $H_{\mathrm{NP}}\,|\emptyset\rangle$ will give a lot of possible double-chain states. Only the ones with lengths equal to those of $|x\rangle$, i.e. states with length $(\frac{1}{2}+2\,x)\,J$ and length $(\frac{1}{2}-2\,x)\,J$, will contribute. All these contributing states can be characterized by a value $\gamma\,J$ expressing how far the cut took place from the straight cut between sites $x\,J$ and sites $(\frac{1}{2}-\,x)\,J$ (see figure 6). Let us denote them $|\{x,\gamma\}\rangle$. The following identity holds:

$$\langle\langle x|H_{\mathrm{NP}}|\emptyset\rangle\rangle = \sum_{i=-\frac{J}{2}}^{\frac{J}{2}} \langle\langle x|\{x,\frac{i}{J}\}\rangle\rangle\,.$$

Overlaps for $\langle x|\{x,\frac{i}{J}\}\rangle$ are shown in figure 7. In order to go to the full cyclic scalar product, one should then add two arbitrary shifts δ and δ' for each piece of $\langle x|$ as well as one for $|\emptyset\rangle$. However, the effect of the latter is simply the multiplication by the factor $l_0\,J$.

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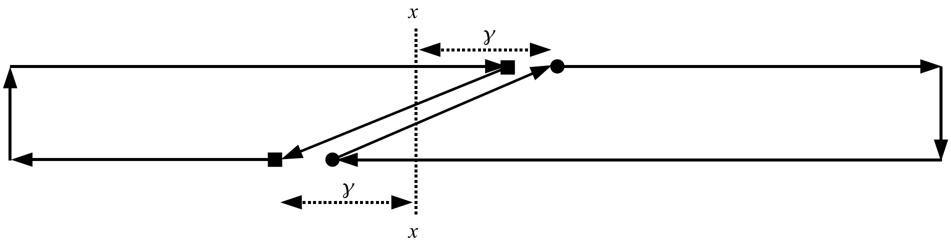

Figure 6: A state $|\{x,\gamma\}\rangle$. Sites at the squares are antisymmetrised, as sites at the circles. The spin chain was cut between sites where θ takes the value $\theta(x+\gamma)$ and $\theta(x-\gamma)$.

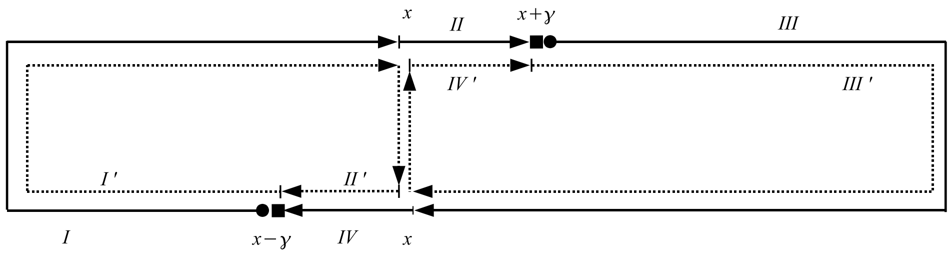

Figure 7: Overlaps between $\langle x|$ (dotted lines) and $|\{x,\gamma\}\rangle$ (continuous lines). Arguments for $\theta(x)$ are given at the relevant points. More precisely, $\langle x|$ reads $\langle I'\ II'|\ \langle\ III'\ IV'|$ while $|\{x,\gamma\}\rangle$ is equal to $|I\ II\rangle\ |III\ IV\rangle$. As in figure 5, it is the function $\theta(x)$ which is continuous along the loop. Possible shifts δ and δ' for each piece of $\langle x|$ were put to 0 for simplicity.

We thus have

$$\langle\!\langle x|H_{\mathrm{NP}}|\emptyset\rangle\!\rangle = l_0 J \sum_{\gamma,\delta,\delta'} \mathcal{F}_{\gamma,\delta,\delta'}\ \langle I'_\delta|I\rangle\ \langle II'_\delta|II\rangle\ \langle III'_{\delta'}|III\rangle\ \langle IV'_{\delta'}|IV\rangle\,,$$

where $\mathcal{F}_{\gamma,\delta,\delta}$ is the anti-symmetrization factor and $\langle I'_\delta|$, $\langle II'_\delta|$, $\left\langle III'_{\delta'}\right|$, $\left\langle IV'_{\delta'}\right|$ are the δ (δ') shifted pieces of $\langle x|$.

Using the approximations we presented at the beginning of this section, we have, for $\gamma > 0$,

$$\begin{aligned}
\langle I'_\delta|I\rangle &\approx \langle \prod_{i=(-\frac{1}{2}-x)\,J}^{x\,J} \overrightarrow{n}_{i-\delta\,J}\ ,\ \prod_{i=(-\frac{1}{2}-x)\,J}^{J} \overrightarrow{n}_{i}\rangle \\
&\approx \exp\left[-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2\right]\,, && (4.21)\\
\langle II'_\delta|II\rangle &\approx \langle\prod_{i=0}^{\gamma\,J}\overrightarrow{n}_{x\,J-i}\,,\prod_{i=0}^{\gamma\,J}\overrightarrow{n}_{x\,J+i}\rangle \\
&\approx \exp\left[-2\,\theta'(x)^2\,J\int_0^\gamma z^2\mathrm{d}z\right] \\
&\approx \exp\left[-\frac{2}{3}\,J\,\gamma^3\,\theta'\,(x)^2\right]\,, && (4.22)
\end{aligned}$$

$$
\begin{aligned}
\langle III'_{\delta'}|III\rangle &\approx \langle \prod_{i=x\,J}^{(\frac{1}{2}-x)\,J} \vec{n}_{i-\delta'\,J}\,,\ \prod_{i=x\,J}^{(\frac{1}{2}-x)\,J} \vec{n}_{i}\rangle \\
&\approx \exp\left[-\frac{1}{2}\,J\,\delta'^2\,\mathcal{E}_2\right]\,, \qquad (4.23)\\
\langle IV'_{\delta}|IV\rangle &\approx \langle \prod_{i=0}^{\gamma\,J} \vec{n}_{x\,J+i}\,, \prod_{i=0}^{\gamma\,J} \vec{n}_{x\,J-i}\rangle \\
&\approx \exp\left[-2\,\theta'(x)^2\,J\int_0^\gamma z^2 \mathrm{d}z\right] \\
&\approx \exp\left[-\frac{2}{3}\,J\,\gamma^3\,\theta'\,(x)^2\right]\,. \qquad (4.24)
\end{aligned}
$$

The overlaps therefore give the contribution

$$
\exp\left[-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2-\frac{1}{2}\,\mathcal{E}_2\,J\,\delta'^2-\frac{4}{3}\,\theta'(x)^2\,J\,\gamma^3\right]\,.
$$

Computing $\mathcal{F}_{\gamma,\delta,\delta'}$ around $\delta=\delta'=\gamma=0$, one gets

$$
\begin{aligned}
\mathcal{F}_{\gamma,0,0} = &\left\langle \vec{n}_{(x-\gamma)\,J+1}\,,\,\vec{n}_{(x+\gamma)\,J}\right\rangle \left\langle \vec{n}_{(x-\gamma)\,J}\,,\,\vec{n}_{(x-\gamma)\,J}\right\rangle \\
&\times \left\langle \vec{n}_{(x+\gamma)\,J}\,,\,\vec{n}_{(x-\gamma)\,J+1}\right\rangle \left\langle \vec{n}_{(x+\gamma)\,J+1}\,,\,\vec{n}_{(x+\gamma)\,J+1}\right\rangle \\
\approx &\ \frac{4}{J}\gamma\,\theta'(x)^2\,. \qquad (4.25)
\end{aligned}
$$

In the $\gamma<0$ case, extra minus signs appear so that one can use the same results by taking the absolute value of γ instead. Using as normalization the factor $\mathcal{N}=\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_0}}\,l_0\,J\right)^{\frac{1}{2}}$ $\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_1}}\,l_1\,J\right)^{\frac{1}{2}}$ $\left(\sqrt{\frac{2\,\pi\,J}{\mathcal{E}_2}}\,l_2\,J\right)^{\frac{1}{2}}$, this leads to

$$
\begin{aligned}
\langle\!\langle x|H_{\mathrm{NP}}|\emptyset\rangle\!\rangle &\approx \frac{1}{\mathcal{N}}\frac{4}{J}\,\theta'(x)^2\,l_0\,J^4\int_{-\infty}^{\infty}\mathrm{d}\gamma\int_{-\infty}^{\infty}\mathrm{d}\delta\int_{-\infty}^{\infty}\mathrm{d}\delta'\ e^{-\frac{1}{2}\,\mathcal{E}_1\,J\,\delta^2-\frac{1}{2}\,\mathcal{E}_2\,J\,\delta'^2\ -\frac{4}{3}\,J\,|\gamma|^3\,\theta'(x)^2}\,|\gamma| \\
&\approx \frac{8\,\Gamma\left(\frac{2}{3}\right)}{3^{1/3}}\,K^{2/3}\,m^{1/3}\,\mathrm{cn}\,(4\,K\,x|m)^{2/3}\left(\frac{l_0}{l_1\,l_2}\right)^{1/2}\left(\frac{2\,\pi\,\mathcal{E}_0}{\mathcal{E}_1\,\mathcal{E}_2}\right)^{\frac{1}{4}}\,J^{-11/12}. \qquad (4.26)
\end{aligned}
$$

This result can be immediately extended to states which were already cut before the action of the Hamiltonian. We note that the non-planar dilatation operator is non-hermitian. A similar situation was encountered in previous analyses of the non-planar corrections to energies of BMN states [19, 10]. There the non-planar dilatation operator was related to its hermitian conjugate by a similarity transformation. The same is the case here.

5. A solvable toy model

By construction the vertically cut multi-string states studied above are degenerate in planar energy with the complete Frolov-Tseytlin string. Let us now consider a toy model of a

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folded string for which the vertically cut states exhaust the space of states degenerate in energy with the uncut string. Furthermore, let us assume that the matrix elements of H_{NP} for string splitting and string joining depend only on the point of splitting and joining. Determining the first non-planar correction to the string energy under these assumptions amounts to diagonalizing the non-planar dilatation operator in the subspace of vertically cut states which of course implies diagonalizing an infinite dimensional matrix in the limit $J \to \infty$. This problem can easily be solved, however. Let us denote by $|i, j, k, \cdots\rangle$ the state corresponding to the string cut at positions $i, j, k, \cdots$ and by $\mathcal{X}_l$ the matrix element corresponding to an additional cut or joining taking place at position l. To illustrate the solution, we consider as an example only three possible sites where a cut/joining can take place. Then in the base $\{|\emptyset\rangle, |1\rangle, |2\rangle, |1,2\rangle, |3\rangle, |1,3\rangle, |2,3\rangle, |1,2,3\rangle\}$, the matrix we have to diagonalize is given by

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{X}_1 & \mathcal{X}_2 & 0 & \mathcal{X}_3 & 0 & 0 & 0 \\ \mathcal{X}_1 & 0 & 0 & \mathcal{X}_2 & 0 & \mathcal{X}_3 & 0 & 0 \\ \mathcal{X}_2 & 0 & 0 & \mathcal{X}_1 & 0 & 0 & \mathcal{X}_3 & 0 \\ 0 & \mathcal{X}_2 & \mathcal{X}_1 & 0 & 0 & 0 & 0 & \mathcal{X}_3 \\ \mathcal{X}_3 & 0 & 0 & 0 & 0 & \mathcal{X}_1 & \mathcal{X}_2 & 0 \\ 0 & \mathcal{X}_3 & 0 & 0 & \mathcal{X}_1 & 0 & 0 & \mathcal{X}_2 \\ 0 & 0 & \mathcal{X}_3 & 0 & \mathcal{X}_2 & 0 & 0 & \mathcal{X}_1 \\ 0 & 0 & 0 & \mathcal{X}_3 & 0 & \mathcal{X}_2 & \mathcal{X}_1 & 0 \end{pmatrix}$$

whose eigenvalues μ are simply all possible sum and differences between the $\mathcal{X}_i$'s:

$$\mu = \pm\mathcal{X}_1 \pm \mathcal{X}_2 \pm \mathcal{X}_3 \ .$$

In the case of J different sites, the eigenvalues are distributed in a quasi-continuum between energies $\pm J \displaystyle\int_{-\frac{1}{4}}^{\frac{1}{4}} \mathcal{X}_x \,\mathrm{d}x$. In our case we can arrange by means of a similarity transformation that all our matrix elements scale as $J^{-5/12}$. Therefore, a rough scaling argument gives

$$\Delta E \approx \frac{\lambda}{N}\frac{J}{2}\mathcal{X}_0 \sim \lambda\, \frac{J^{7/12}}{N}. \tag{5.1}$$

Now if one, again naively, assumes BMN-like scaling for the energy of spinning strings one needs that the genus one contribution compared to the genus zero one has an additional factor of $\frac{J^2}{N}$ which leads to the expectation $\Delta E \sim \frac{J}{N}$. It is of course not known to which extent BMN scaling beyond the planar limit should hold for spinning strings. One knows from the analysis of [20, 21] and the field theoretical computations of [22] that BMN scaling for few-impurity operators breaks down already at the planar level but only at order four in λ. In the true picture of string splitting we can not claim that the straight cut states exhaust the space of eigenstates degenerate in energy with the folded string.[5] One could argue that one should in fact replace $\mathcal{X}_x$ of the toy model by some integral over matrix

[5] As mentioned earlier the straight cut states are also not exact eigenstates but only eigenstates up to terms of order $\frac{1}{J}$.

elements involving skew cut states close to the vertically cut ones and that this could give rise to additional factors of J. We have not been able to make a quantitative estimate of this effect, but we find it unlikely that such an integration could provide the "missing" factor $J^{5/12}$. Rather the low power of J in eq. (5.1) seems to suggest that the process of semi-classical string splitting and joining is not of importance for the genus one energy shift, cf. section 6.

6. Discussion

Our calculation shows that for long strings a nonzero contribution to the splitting matrix element comes only from strings which are almost on top of each other, cf. eq. (4.12) and subsequent calculations. This is somewhat reminiscent of the interaction vertex between strings in light cone string field theory:

$$\begin{aligned} & V(X_0^i(\sigma), X_1^i(\sigma), X_2^i(\sigma)) = \\ & \int ds_0 ds_1 ds_2\, \delta(J_0 - J_1 - J_2) \times \\ & \prod \Delta(X_1^i(\sigma + s_1) - X_0^i(\sigma + s_0))\Delta(X_2^i(\sigma + s_2) - X_0^i(\sigma + s_0 + J_1/J_0)). \end{aligned} \tag{6.1}$$

In the above formula the s_i are direct analogues of cyclic translations in our definition of states, while the functional delta functions are analogues of the property that we have found namely that in order for the matrix element to be nonzero the angles defining the coherent states have to be within $J^{-1/2}$. However the detailed calculations in sections 4.2 and 4.3 show that more nontrivial $J^{-1/3}$ factors may also appear. In addition we saw that the H_{NP} operator gives an effective additional operator inserted at the interaction point, cf. eqn (4.25). This is not unexpected since such operators appear generically in superstring light cone SFT (see e.g. [23]). However, due to the fact that we really can deal only with classical states we refrain from making any more quantitative comparison.

Our crude estimate of the order of magnitude of the genus one energy shift due to semi-classical string joining and splitting leads to the energy scaling with an unexpectedly small power of J. An interpretation of this result may be that the contribution to the energy shift coming from such semi-classical string processes is simply quite small. In fact for generic macroscopic rotating strings (i.e. not 'folded' ones) the contribution of string splitting into classical states would be very strongly suppressed. It is much more probable that the dominant non-planar contribution would come from *small* strings which would split off from the rotating string and which would be reabsorbed shortly after. Unfortunately the process of small strings splitting off is beyond the reach of the semi-classical coherent state methods which we were using.

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